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NONLINEAR EXOGENEOUS SYSTEM AND INTERNAL MODELS DESIGN

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The problem of output regulation of plants affected by nonlinear exogenous systems is addressed. The new approach of designing the controller having an internal model, which repeats the dynamics of the disturbance is presented. Two realizable controllers which using measurements of the output and guarantees the achievement of a control goal are proposed: the one for known parameters and the adaptive one.

Keywords: *control systems, nonlinear systems, adaptive systems, internal model*

Introduction. In this paper we present the new approaches of internal model design for output regulation problems with nonlinear exogenous systems. The problem of output regulation for nonlinear systems has been studied in several works; the monograph [1] shows a background. The main idea is to design the controller having an internal model which repeats the dynamics of the disturbance [2, 3] or tends to repeat with adaptive tuning of critical parameters of exogeneous system [4]. While for such systems the theory of linear regulation is well established [3], a fully satisfactory nonlinear version does not seem to have been proposed yet [5—8]. In the recent paper [9], we have shown how a result of this kind can be achieved in the case of linear systems. In the paper [10] an extension of such results to the case of nonlinear systems affine in the control input is presented.

Problem formulation. Consider the simple plant

$$\dot{y} = u + \delta(w) \tag{1}$$

driven by the control u and the disturbance δ, which is the output of nonlinear exogeneous system

$$\dot{w} = s(w, \theta) \tag{2}$$

with the state $w \in \mathbb{R}^2$, nonzero initial conditions $w(0)$ and some constant, possibly unknown, parameter $\theta < 0$, and the following view of the functions s and δ:

$$s_1(w) = w_2,\ s_2(w) = \theta w_1^3,\ \delta(w) = w_1$$

or

$$\ddot{\delta} = \theta\delta^3. \tag{3}$$

The goal is to design the control u in order to provide asymptotic convergence of the output y to 0:

$$\lim_{t\to\infty} y(t) = 0. \tag{4}$$

Internal model design. The motivation of this study is to find new approaches of internal models design for output regulation problems with nonlinear exogenous systems.

The main idea is to design the controller having an internal model, which repeats the dynamics of the disturbance without trying to find functions $\sigma(w)$ and $\gamma(\sigma)$, which satisfies

$$\frac{\partial\sigma(w)}{\partial w} s(w) = F\sigma(w) + G\psi(w), \tag{5}$$

$$\psi(w) = \gamma(\sigma(w)), \tag{6}$$

where a pair (F,G) is in canonical controllable form with Hurwitz F, the function $\psi(w)$ is a steady state of the control input ensuring the identity

$$0=\psi(w)+\delta(w). \tag{7}$$

However, it is quite difficult to find analytically appropriate functions $\delta(w)$ and $\gamma(\sigma)$.

Let the dynamics of the system be augmented by a chain of two integrators

$$u=\eta_1,\ \dot{\eta}_1=\eta_2,\ \dot{\eta}_2=v,$$

in which v is a new input, to be designed.

Define

$$\xi_1=y,\ \xi_2=\eta_1+\delta,\ \xi_3=\eta_2+\dot{\delta},$$

so that

$$\dot{\xi}_1=\xi_2,\ \dot{\xi}_2=\xi_3,\ \dot{\xi}_3=v+\ddot{\delta}=v+\theta\delta^3,$$

i.e.

$$\dot{\xi}=F\xi+G\left[v+\theta\delta^3\right],$$

in which F, G are in “prime form”, i.e.

$$F=\begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix},\quad G=\begin{pmatrix}0\\0\\1\end{pmatrix}.$$

The nominal feedback-linearizing control

$$v_{\text{nom}}=-\theta\delta^3+K\xi$$

yields a system

$$\dot{\xi}=(F+GK)\xi,$$

in which K can be chosen so that $(F+GK)$ is Hurwitz.

Such control is not implementable. Instead, observing that $\delta=\xi_2-\eta_1$ we pick

$$v=-\theta\,\text{sat}\left[\left(\hat{\xi}_2-\eta_1\right)^3\right]+K\hat{\xi},$$

in which $\text{sat}(\cdot)$ is a saturation function and $\hat{\xi}$ is generated by the observer

$$\begin{aligned}\dot{\hat{\xi}}_1&=\hat{\xi}_2+\kappa a_2\left(y-\hat{\xi}_1\right),\\ \dot{\hat{\xi}}_2&=\hat{\xi}_3+\kappa^2 a_1\left(y-\hat{\xi}_1\right),\\ \dot{\hat{\xi}}_3&=\kappa^3 a_0\left(y-\hat{\xi}_1\right)\end{aligned} \tag{8}$$

with some positive design parameters a_0,a_1,a_2. This control is implementable, because η_1 is available for feedback.

As result, we get

$$\dot{\xi}=(F+GK)\xi+G\Delta_1,$$

in which

$$\Delta_1=K\left(\hat{\xi}-\xi\right)+\theta\left[\delta^3-\text{sat}\left[\left(\hat{\xi}_2-\eta_1\right)^3\right]\right].$$

Define the errors as usual:

$$e_1 = \kappa^2\left(\xi_1 - \hat{\xi}_1\right),\quad e_2 = \kappa\left(\xi_2 - \hat{\xi}_2\right),\quad e_3 = \xi_3 - \hat{\xi}_3 \tag{9}$$

and compute its derivatives

$$\dot{e}_1 = -\kappa a_3 e_1 + \kappa e_2,\quad \dot{e}_2 = -\kappa a_2 e_1 + \kappa e_3,\quad \dot{e}_3 = -\kappa a_1 e_1 + K\xi + \Delta_1. \tag{10}$$

Rewrite (10) in matrix form

$$\dot{e} = \kappa A e + B\left[K\xi + \Delta_1\right],$$

where

$$A = \begin{pmatrix} -a_2 & 1 & 0 \\ -a_1 & 0 & 1 \\ -a_0 & 0 & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Let us pick numbers a_0, a_1, a_2 such that the matrix A is Hurwitz.

Note that $\hat{\xi}_2 - \eta_1 = -\frac{e_2}{\kappa} + \xi_2 - \eta_1 = -\frac{e_2}{\kappa} + \delta$. Hence,

$$\Delta_1 = -K\begin{pmatrix} \kappa^{-2} & 0 & 0 \\ 0 & \kappa^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + \theta\left[\delta^3 - \operatorname{sat}\left[\left(\delta - \frac{e_2}{\kappa^2}\right)^3\right]\right]$$

that we rewrite as

$$\Delta_1(e,\delta) = M(\kappa)e + \theta\Gamma(e,\delta).$$

If $\kappa > 1$, then $M(\kappa) = K$.

If initial conditions for the exogenous variable w lie in a compact (invariant) set (which is a reasonable assumption), we have $|\delta(t)| \le \delta_0$ for some δ_0. Thus, if the threshold of the saturation is larger than δ_0^3, we have $\Gamma(0,\delta)$=0. Finally, since δ is bounded and the saturation function is bounded, we have that also $\Gamma(e,\delta)$ is bounded. In other words, $\Gamma(e,\delta)$ is a bounded function that vanishes at $e = 0$. We can take advantage of such properties in the subsequent analysis.

The full dynamics of the closed-loop system may be described with the state vector $x = \begin{pmatrix} \xi \\ e \end{pmatrix}$, which satisfies the following equation:

$$\begin{pmatrix} \dot{\xi} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} (F + GK)\xi + GM(\kappa)e + G\theta\Gamma(e,\delta) \\ \kappa A e + B\left[K\xi + M(\kappa)e + \theta\Gamma(e,\delta)\right] \end{pmatrix},$$

where both $(F + GK)$ and A are Hurwitz.

From this point on the analysis can proceed using the small-gain theorem. In fact, in the lower equation, the term $\Gamma(e,\delta)$, which is bounded function vanishing at $e = 0$, can be bounded as $|\Gamma(e,\delta)| \le Ne$, for some N. Hence, by increasing κ one can arbitrarily lower the "gain" between the "input" ξ and the state e. The upper equation, in turn, viewed as a system with state ξ and input e, is an input-to-state stable system, with a fixed gain. From this, by standard arguments it is deduced that if κ is large enough, the equilibrium point $(\xi, e) = (0,0)$ is globally stable.

Remark. *The problem could have been addressed also with the approach of [8], but we are now proposing a different approach, because we think that this is more convenient to address the case of uncertain* θ *(the approach of the paper [8] presumed accurate knowledge of the exogenous system).*

Adaptive output regulation. In this section we will show the realizable adaptive controller which using measurements of the output y only allows to achieve the goal (4) without knowledge of θ. Consider the control law with an adaptive internal model

$$u = \eta_1,$$
$$\dot{\eta}_1 = \eta_2, \quad \dot{\eta}_2 = \hat{v},$$
$$\hat{v} = -\hat{\theta}\mathrm{sat}\left[\left(\hat{\xi}_2 - \eta_1\right)^3\right] + K\hat{\xi}, \tag{12}$$
$$\dot{\hat{\theta}} = \mu y \mathrm{sat}\left[\left(\hat{\xi}_2 - \eta_1\right)^3\right],$$

where $\mu > 0$ is a positive number, $\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3$ are the states of the observer of the form (8).

The simulation results of such adaptive feedback are given in the next section.

Simulations. In this section we show the results of simulation for plant (1)—(3) to achieve the goal (4). The control law with an adaptive internal model (12) was used to stabilize the plant with an external disturbance. Fig. 1 shows transients for the output controller with known θ and different κ, as well as disturbance signal. Fig. 2 shows transients for the output controller with known and different θ; this parameter affects exogenous signal as shown. Fig. 3 and 4 show the output controller and estimation of θ under different values of parameters. The robustness of proposed control law shown under different conditions.

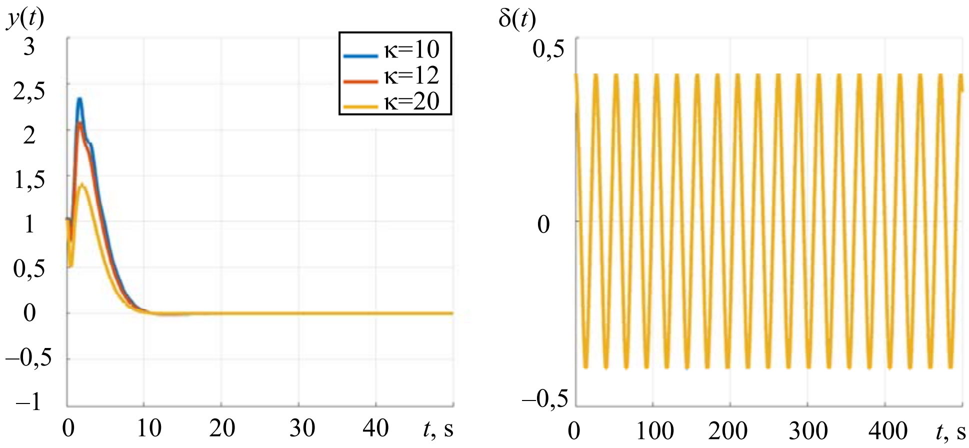

Fig. 1

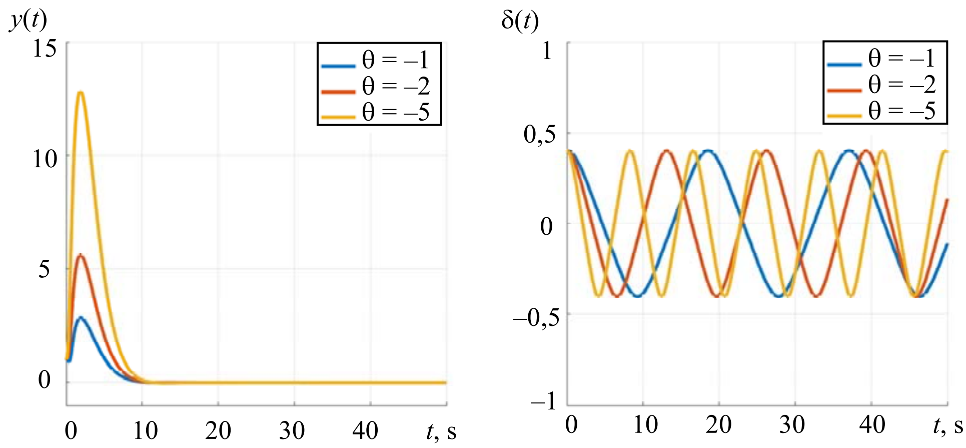

Fig. 2

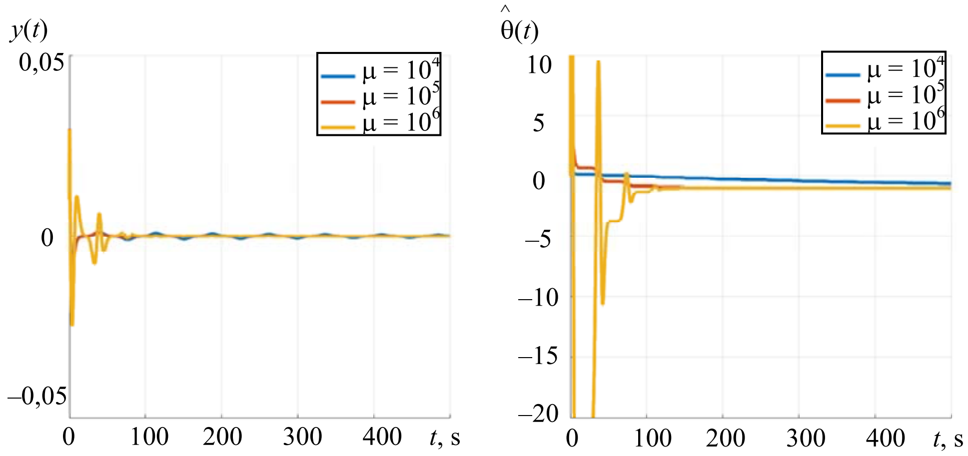

Fig. 3

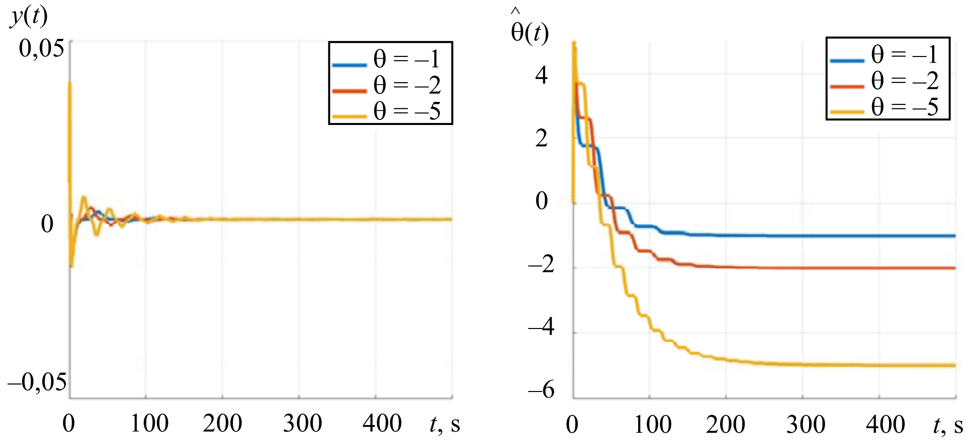

Fig. 4

Conclusion. In this paper the new approach of internal model design for output regulation problem with nonlinear exogenous systems is studied. The main idea is to design the controller having an internal model, which repeats the dynamics of the disturbance. The realizable controller which using measurements of the output and guarantees the achievement of a control goal is proposed. Also, the realizable adaptive controller which using measurements of the output without knowledge of disturbance parameters provided as well. The work is in progress to show how some issues left open in the current presentation can be addressed.

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НЕЛИНЕЙНЫЕ ЭКЗОГЕННЫЕ СИСТЕМЫ И ПОСТРОЕНИЕ ВНУТРЕННЕЙ МОДЕЛИ

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Рассматривается задача управления по выходу объектами, подверженных воздействиям нелинейных экзогенных систем. Представлен новый подход построения регулятора, содержащего внутреннюю модель, который повторяет динамику внешних воздействий. Предложено два реалиизуемых регулятора, использующих измерения выхода объекта и гарантирующих достижимость цели управления: один для известных параметров и один адаптивный регулятор.

Ключевые слова: системы управления, нелинейные системы, адаптивные системы, внутренняя модель

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